PCA Principal Component Analysis

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MTech CSE



OVERVIEW

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- 5. Useful properties of PCA
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INTRODUCTION

Central idea of principal component analysis:

- reduce of dimensionality of a data set consisting of large number of interrelated variables
- retaining as much as possible of the variation present in the data set.
- This is achieved by transforming to a new set of variables, principal components(PCs)
- Pcs are uncorrelated and ordered so that the first few retains the most of the variation present.

PCA finds a linear projection of high dimensional data into a lower dimensional subspace such as:

○ The variance retained is maximized.(??)

PRE-REQUISITES

Some of the pre requisites:

- 1. Eigen vectors
 - 1.1 If A is a square matrix then a non zero vector v is an eigenvector of A if there is a scalar quantity such that

 $Av=\lambda v$

DERIVATION

PCA - DERIVATION

Given a zero-mean random variable

 $X \in \mathbb{R}^D$,

the first principal component of X is defined as

 $v_1 = argmax_{v1}var(Xv_1)$

Subject to :
$$v_1^T v_1 = 1$$

Rewrite in terms of the covariance matrix:

$$var(Xv_1) = \frac{1}{N-1}(Xv_1)^T(Xv_1) = v_1^T Cv_1$$

Solved via constrained optimisation using Lagranges method:

$$L(v_{1}, \lambda_{1}) = v_{1}^{T} C v_{1} + \lambda_{1} (1 - v_{1}^{T} v_{1})$$

PCA - DERIVATION CONTD...

Constrained Optimisation:

$$L(v_{1}, \lambda_{1}) = v_{1}^{T} C v_{1} + \lambda_{1} (1 - v_{1}^{T} v_{1})$$

Gradient with respect to v:

$$\frac{dL(v_1,\lambda_1)}{dv_1} = 2Cv_1 - 2\lambda_1v_1 \Longrightarrow Cv_1 = \lambda_1v_1$$

This is the eigenvector problem! Multiply by v1T:

$$\lambda_1 = v_1^T C v_1$$

The projection variance is the eigenvalue And further we can find more PCs.

PCA - STEPS TO BE FOLLOWED

PCA Steps: Transform an N x d matrix into an N × m matrix :

- Centralized the data (subtract the mean).
- Calculate the × covariance matrix:

$$C = \frac{1}{N-1} X^T X$$

- Calculate the eigenvectors of the covariance matrix.
- Select m eigenvectors that correspond to the largest m eigenvalues to be the new basis.

USEFUL PROPERTIES OF PCA

PCA Steps: Transform an N x d matrix into an N × m matrix :

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GPCA - GENERALIZED PRINCIPAL COMPONENT ANALYSIS

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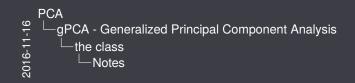
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