Quadratic Functions

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Mathematics Standard Level

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1 Introduction

Quadratic Functions

1.1 What are they?

Quadratic Functions are functions in the form of $f(x) = ax^2 + bx + c$. Where variables a, b, nor c can equal to 0.

Here are some examples of Quadratic Function that we will look at:
1.$f(x) = x^2 - 7x + 12$

2.$f(x) = 4x^2 + 20x + 25$

3.$f(x) = -3x^2 - 5x + 7$

1.2 How do we solve these Quadratic Functions?

There are many ways to solve Quadratic Functions but we will only focus on two famous methods:

1. **Quadratic Formula**

2. **Graphing Method**

2 Quadratic Equation

2.1 What is a Quadratic Equation?

Quadratic Equation is an equation used to solve any Quadratic Function. We can follow just a formula to get the answer for x.

The formula

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ when $ax^2 + bx + c = 0$
Using this formula allows us to get the answer quickly.

2.2 How do we use it?

A great Question. Well we basically see the Quadratic Function ($ax^2 + bx + c$).

Then we just match the letters with the quadratic formula, so we place the **A** from $\underline{A}x^2$ to the a from the Quadratic Equation. The same rule applies to **B** from $\underline{B}x$ and **C** from $\underline{c}$.

2.3 How can we be sure this formula works?

Here is the proof that supports this formula:
$ax^2 + bx + c = 0$ − **Step** 1
$ax^2 + bx = -c$ − **Step 2**
$x^2 + \frac{bx}{a} = \frac{-c}{a}$ − **Step 3**
$x^2 + \frac{bx}{a} + \frac{b}{2a}^2 = \frac{-c}{a} + \frac{b}{2a}^2$ − **Step 4**
$(x + \frac{b}{2a})^2 = -\frac{c}{a} + \frac{b^2}{2a^2}$ − **Step 5**
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$ − **Step6**
$x + \frac{b}{2a} = \frac{\sqrt{b^2-4ac}}{2a}$ − **Step 7**
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ − **Step 8**
This helps us understand how the quadratic equation was formed thus gives us a good insight to trusting this formula.

2.4 Lets solve :D

1.$f(x) = x^2 - 7x + 12$

Step 1. Lets identify the **A,B, and C**

In this case x^2 does not have a coefficient in front of it, so therefore there must be a 1 beside x^2. Thus the value for **A** is going to be 1.

Then the coefficient beside x is -7, thus the value for **B** becomes -7
Finally C, C is going to be 12.

$A = 1, B = -7, C = 12$

Step 2. Time to use the quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So we can plug in the variables

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(12)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{1}}{2}$$

$$x = \frac{7 \pm 1}{2}$$

Therefore $x = 4/3$ This is supported by the fact that:
$x = \frac{8}{2}$=4 which was created by $x = \frac{(7+1)}{2}$

and also

$x = \frac{6}{2}$=3 which was created by $x = \frac{(7-1)}{2}$

2.$f(x) = 4x^2 + 20x + 25$

Step 1. Lets identify the **A,B, and C**

In this case, $\underline{4}x^2$, the 4 is going to be the variable **A**.
Variable B is going to be 20 since $\underline{20}x$ has the coefficient is 20.
Variable c is going to be 25.

$A = 4, B = 20, C = 25$

Step 2. Using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now we just plug in the values:

$$x = \frac{-(20) \pm \sqrt{(20)^2 - 4(4)(25)}}{2(4)}$$

$$x = \frac{-20 \pm \sqrt{0}}{8}$$

$$x = \frac{-20 \pm 0}{8}$$

Therefore $x = -2.5$

Since $\frac{-20}{8} = -2.5$ which was got by $\frac{-20+0}{8}$ and $\frac{-20-0}{8}$. Both these equations equal the same answer.

$$3. f(x) = -3x^2 - 5x + 7$$

Step 1. Find the variable **A,B,C**

$$3. f(x) = -3x^2 - 5x + 7$$

So Variable **A** is going to be -3 since -3 is in $\underline{-3}x^2$. Variable is B is going to be -5 since -5 is in $\underline{-5}x$. Finally, Variable **C** is going to be 7.

$A = -3, B = -5, C = 7$

Step 2. Using the formula :D

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Plug in the variables:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-3)(7)}}{2(-3)}$$

$$x = \frac{5 \pm \sqrt{109}}{-6}$$

$$x = \frac{5 \pm 10.4}{-6}$$

Therefore $x = -2.5$ or .9
We got this from doing this:

$$x = \frac{15.4}{-6} = -2.5$$

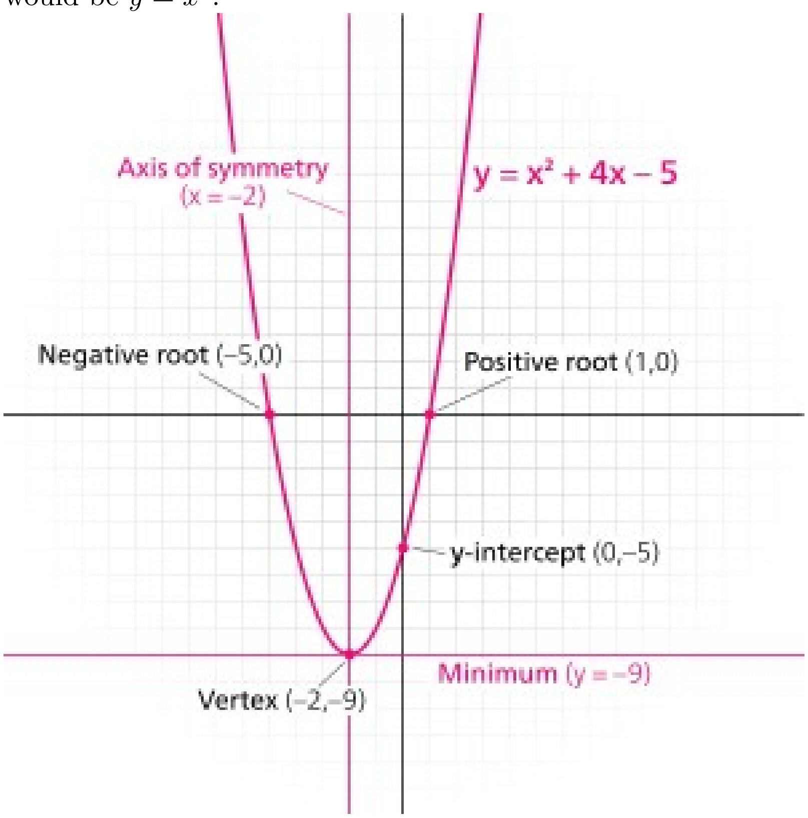
$$x = \frac{-5.4}{-6} = 0.9$$

3 What can we do with Quadratic Functions?

Quadratic Functions are really useful as they create parabolas when graphed.

3.1 Parabola's

A parabola is U shaped figure that forms when graphed by a quadratic function. The most common Quadratic Function that draws a parabola would be $y = x^2$.



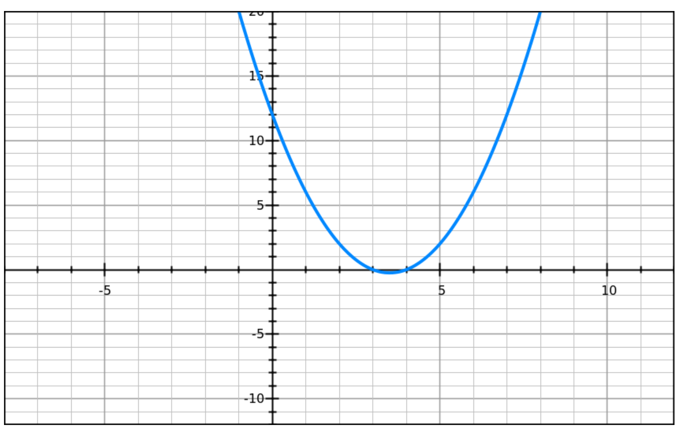
3.1.1 So what's so good about parabola's

Parabola's play an important part in our architecture lives, an example of this would be bridges. Bridges, itself is a parabola, we need bridges to be a parabola because parabola's are the most strongest shape that can hold lots of cars.
Here are some other examples of parabola's:
1. Car headlights
2. Heaters
3. Satellite Dishes
4. Even the McDonald Arch
Etc. Parabola's also form in nature, an example of this would be water coming out of water fountains,they form a parabola. Even when you throw a ball it forms a parabola (Projectile).

3.2 Let's see how to do graph these functions?

Equation 1 :

$f(x) = x^2 - 7x + 12$



Step 1. We need to find the axis of symmetry

Axis of Symmetry Formula:

$$x = \frac{-b}{2a}$$

Step 2. Plug in the values
a= 1 and b= -7

$$x = \frac{-(-7)}{2(1)}$$

$$x = \frac{7}{2}$$

$$x = 3.5$$

x is going to be 3.5

step 3. Plug in x into the original functions

$$f(x) = x^2 - 7x + 12$$

$$f(3.5) = (3.5)^2 - 7(3.5) + 12$$

$$f(3.5) = 12.3 - 24.5 + 12$$

$$f(3.5) = (-0.2)$$

Step 4. Identifying the vertex

Now we have both the x and y value, we can now find the vertex.

$$(x, y) = (3.5, (-0.2))$$

SO far, we know the
axis of symmetry which is $x = 3.5$
vertex which is $(3.5, -0.2)$
We still need to find out the x-intercept.

x-intercept
we can find this out when $y = 0$
We can look at the graph and see that there are two x-intercepts, one is 3 and the other is going to be 4

This matches with our answer in which we used the quadratic equation to find.

If we wanted to find the x value with a given y value, we can do that as well. Say for example we wanted to find:

$$f(x) = 6$$

Using the graph for this equation, we can see that the y value of 6 has two x values: 1 and 6.

But our eyes may not be accurate enough, let's use math and see what the real value of x is. We simply replace the $f(x)$ with 6 in the original equation as they are equal to each other.

$$6 = x^2 - 7x + 12$$

Now it's simple algebra

$$0 = x^2 - 7x + 6$$

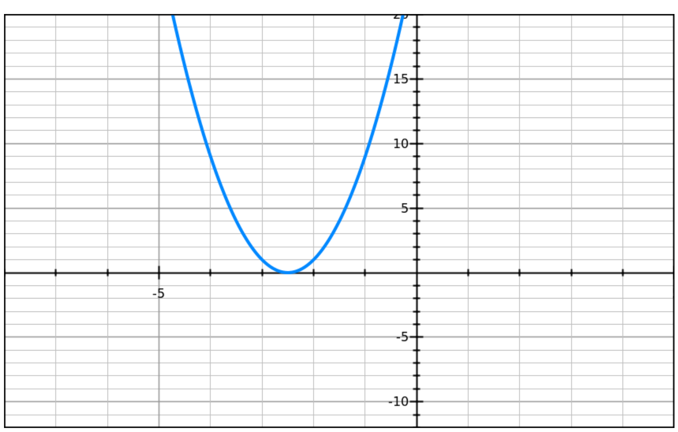
$$0 = (x - 6)(x - 1)$$

Therefore:

$$6 = f(6), f(1)$$

Equation 2.

$$f(x) = 4x^2 + 20x + 25$$



We got this graph by first finding out the axis of symmetry

$$x = \frac{-b}{2a}$$

Step 2. Plug in the values since a= 4 and b=20

$$x = \frac{-(20)}{2(4)}$$

$$x = \frac{-20}{8}$$

$$x = -2.5$$

Step 3. Plug in x

$$f(-2.5) = 4(-2.5)^2 + 20(-2.5) + 25$$

$$f(-2.5) = 25 + (-50) + 25$$

$$f(-2.5) = 0$$

So y=0.

x-intercept

We already found out x-intercept since y=0, so therefore -2.5 is going to be the x-intercept.

What is $f(x) = 9$?

Same as above, we replace $f(x)$ with 9

$$9 = 4x^2 + 20x + 25$$

$$0 = 4x^2 + 20x + 16$$

$$0 = 4x^2 + 4x + 16x + 16$$

$$0 = 4x(x+1) + 16(x+1)$$

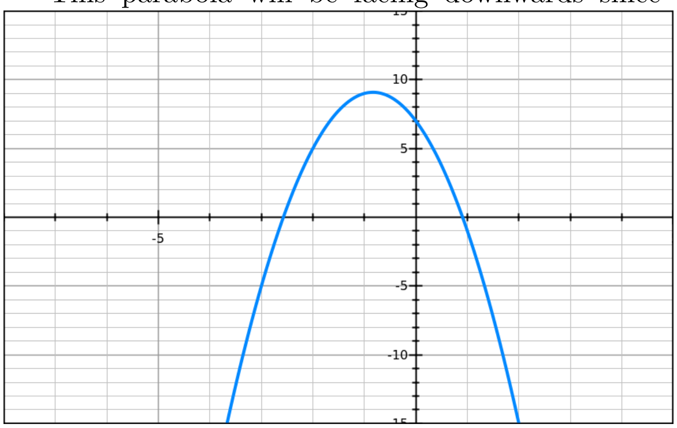
$$0 = (4x+16)(x+1)$$

$$0 = 4(x+4)(x+1) = (x+4)(x+1)$$

Therefore:

$$9 = f(-4), f(-1)$$

Equation 3
$f(x) = -3x^2 - 5x + 7$
This parabola will be facing downwards since x^2 has a - coefficient



We got this graph by first finding out the vertex.

$$x = \frac{-b}{2a}$$

A=−3 and B=−5

$$x = \frac{-(-3)}{2(-5)}$$

$$x = \frac{3}{-10}$$

$$x = -0.3$$

Now we use the axis of symmetry value to find out the f(x)

$$f(x) = -3x^2 - 5x + 7$$

$$f(x) = -3(-0.3)^2 - 5(-0.3) + 7$$

$$f(x) = -0.3 - (-1.5) + 7$$

$$f(x) = 8.2$$

The vertex for this graph is going to be $(-0.3, 8.2)$

Now we can graph this equation The parabola is going going downwards since the first coefficient is a negative number and thus the parabola is going downwards.

The x-intercept is going to be (-2.5) and (0.9)

Finally, let's see what $f(x) = 7$ is:

$$7 = -3x^2 - 5x + 7$$

$$0 = -3x^2 - 5x + 0$$

$$0 = -3x^2 - 5x$$

In this case, we can substitute x with 0 to get 0.

Therefore

$$7 = f(0)$$

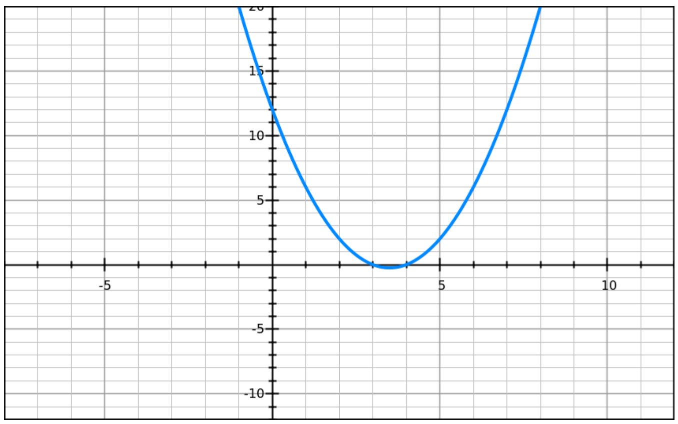
3.2.1 Lets see how this graph works

We can see how this works by plugging a number for f(x) and we can see the answer.

Equation 1

1.$f(x) = x^2 - 7x + 12$

So let f(x)=3, we look at the number 3 on the y-Axis

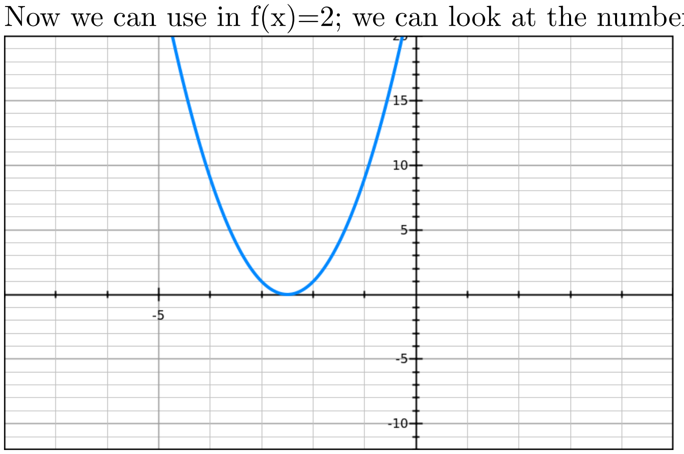


We can see that 5.3 and 1.69 came up from the x-axis. That is the answer

Equation 2

$$2. f(x) = 4x^2 + 20x + 25$$

Now we can use in f(x)=2; we can look at the number 2 on the y-axis.



We can see that the x value needed to make sure that f(x)=2 can be either -1.79 or -3.2.

How does this work?

There seems to be two answers and not just one? Well it's because that a parabola is a special case and has 2 roots not just one.

Equation 3

$3. f(x) = -3x^2 - 5x + 7$

Conclusion

Quadratic functions can be seen everywhere in nature; from the arc of a fountain to the flight time of a ball thrown up into the air. Any object with parabolic properties will have a quadratic function related to it. A function where one input spits out two outputs, quadratic functions are truly unique and one of a kind.