Saturated Nuclear Matter in the Large N_c and Heavy Quark Limits of Quantum Chromodynamics

Ishaun Datta

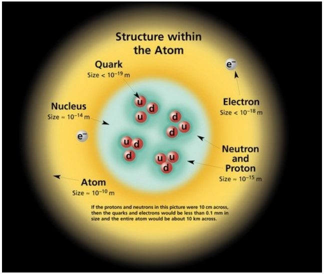
Montgomery Blair High School
Silver Spring, MD

Google Science Fair 2014

Outline

- Background
  - Subatomic particles
  - What is Quantum Chromodynamics (QCD)?
  - QCD Intractability
- N_c and M_q Limits: A Tractable Approach
- Saturated Nuclear Matter
  - Prior research: Leading order
  - New considerations: Sub-leading order
- Calculations and correction terms
  - Nearest-neighbor distance
  - Crystal structure
- Conclusions
- Acknowledgments

- Nuclei consist of protons and neutrons, which are hadrons
- Hadrons are particles whose dominant interaction is the **strong** nuclear force



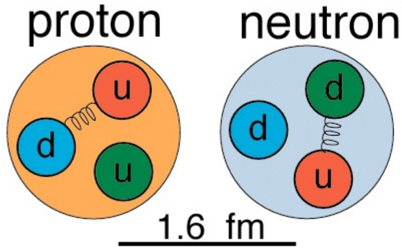


Figure : Left: Subatomic structure from AP Chemistry Wikispaces
Right: PBS portrayal of up and down quarks that make up nucleons

Quantum Chromodynamics (QCD)

- Quantum field theory of the strong interaction
  - Governs nuclear physics
- QCD is a theory of quarks and gluons
  - Quarks are sub-nuclear particles carrying “color”
    - Colors are red, green, and blue
    - Objects must be in a color-singlet state, i.e. color-neutral
  - “Gluons” mediate the strong interaction

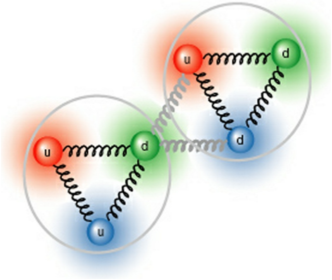


Figure : Gluons are what keep protons in a nucleus together. Image courtesy of ScienceBlogs.com.

QCD Intractability

In principle, one could extract all useful information from the QCD Lagrangian. In practice:

- Perturbative expansions fail to converge
- Numerical techniques
  - Lattice QCD numericizes path integrals
  - Rapidly oscillating functions prevent accurate results

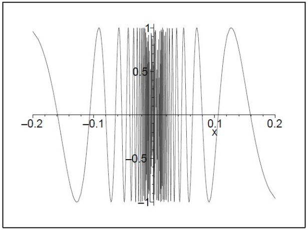


Figure : $\sin(\frac{1}{x})$ is an example of an oscillatory function that would be impossible to integrate numerically.

The N_c and M_q Limits of QCD

N_c : number of colors.
M_q : quark mass.
When N_c and M_q grow arbitrarily large, QCD simplifies greatly.
This model is based on expansions in powers of $\frac{1}{N_c}$.

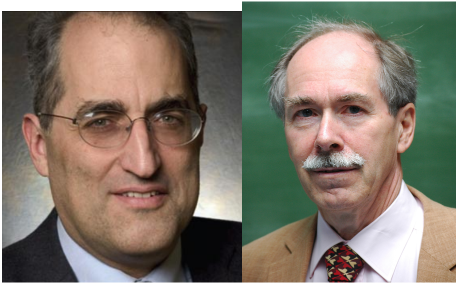


Figure : Edward Witten (left) and Gerard 't Hooft (right), pioneers of the double limit of QCD. Images from the Institute for Advanced Study.

The N_c and M_q Limits of QCD, continued

Important to note:

- Hadrons have two sub-groups
  - Baryons (three quarks) and mesons (quark-antiquark)
  - Mesons are unstable and thus unimportant to this research

Simplifications of the double limit:

- Reduces a field theory to a quantum mechanical theory to describe baryons
- Particles move nonrelativistically
- The strong force reduces to a color-Coulomb interaction

The Problem: Saturated Nuclear Matter

- Does nuclear matter bind to itself in the double limit?
  - Self-binds, i.e. saturates in the real world
  - Previous research: repulsive at leading order
- Sub-leading order considerations
  - Sub-leading order usually unimportant
  - "Glueballs," conglomerate particles of color-carrying gluons, provide an attractive force at sub-leading order

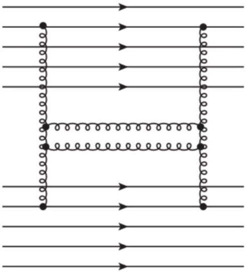


Figure : Two interacting gluons must exchange at least two quarks to preserve a color singlet, giving rise to the "glueball."

Methods

- Analytic and numerical components
  - Analytic work completed by hand
  - Wolfram Mathematica software used to generate numerical solutions
- The two approaches produced results that agree closely

Figure : Mathematica Icon, courtesy of Wolfram.com.

At leading order, the interaction energy per baryon is entirely Pauli repulsion:

$$E_{\text{Pauli}} = c_1 N_c M_q \tilde{\alpha}_s{}^2 \tilde{d}^p \exp(-c_2 \tilde{d}), \qquad (1)$$

where c_1, c_2, and p are known numerical constants and $\tilde{d} = (\tilde{\alpha}_s m_q d)$. The variable d is the distance between nearest-neighbor baryons.

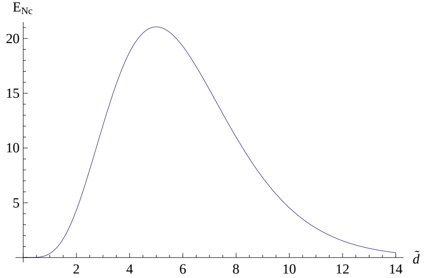


Figure : Leading order interaction energy vs. separation distance.

Sub-leading order

According to quantum field theory, point sources have a Yukawa potential:

$$V(r) = -a\frac{e^{-kmr}}{r}, \tag{2}$$

where r is the distance to the particle, m is the mass of the particle, and a and k are constants.
The potential for a system of n_{gb} glueballs is

$$V_{\text{glueball}} = \sum_{i=1}^{n_{gb}} -a\frac{\exp(-km_i r)}{r} \tag{3}$$

Taking m_1 to be the least massive glueball and factoring:

$$V = -a\frac{e^{-km_1 r}}{r}[1 + \sum_{j=1}^{n_{gb}} \frac{\exp(-km_i r)}{\exp(-km_1 r)}] \tag{4}$$

So all but the lightest glueball is exponentially suppressed.

Sub-leading order, continued

Glueball potential is re-expressed as

$$E_{gb} = -\tilde{g}_{gb} \frac{\exp(-\tilde{m}_{gb}\tilde{d})}{\tilde{d}} \tag{5}$$

where $\tilde{m}_{gb} \equiv \frac{m_{gb}}{\tilde{\alpha}_s M_q}$ and $\tilde{g}_{gb}$ is a dimensionless constant.

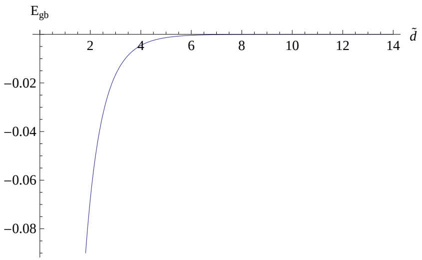


Figure : Glueball interaction energy vs. separation distance.

Nuclear Matter Saturates in the Double Limit

Interaction energy at sub-leading order in N_c:

$$E_{\text{Interaction}} = E_{\text{Pauli}} + E_{\text{gb}} \tag{6}$$

Differentiating yields separation distance between baryons:

$$\tilde{d} \approx \frac{1}{(c_2 - \tilde{m}_{\text{gb}})} \ln\left(\frac{c_1 N_c M_q \tilde{\alpha}_s}{\tilde{g}_{\text{gb}}}\right) \tag{7}$$

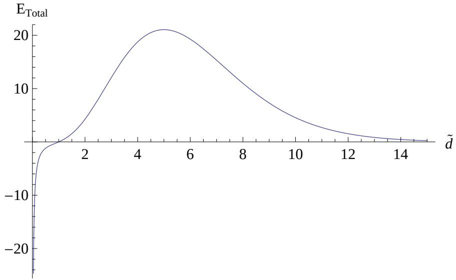


Figure : Total interaction energy vs. separation distance. Saturation occurs at the x-intercept.

The derivation of the nearest-neighbor distance used the approximation $\ln \tilde{d} << \tilde{d}$.

Error terms are of the form:

$$\ln(\ln(\frac{c_1 N_c M_q \tilde{\alpha}_s}{\tilde{g}_{\mathrm{gb}}})) \tag{8}$$

Note that $\ln \tilde{d}$ converges so slowly as to be a constant for all intents and purposes.

Analytic and Numerical Agreement

Analytic and numerical values for $\tilde{d}$ agree in the large N_c limit.

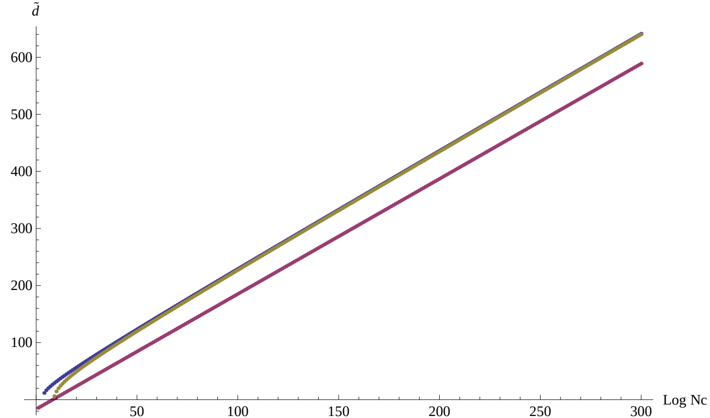
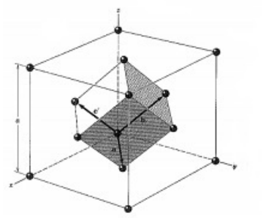


Figure : The analytic approximation (red) is approximately linear in $\log N_c$ and parallel to the numerical calculation (blue). The analytic approximation with the correction term (green) is indistinguishable from the numerical calculation for large N_c.

Crystal Structure

- Baryons form a crystal in the M_q limit
- Lowest energy configuration corresponds to maximal atomic packing fraction
- Greatest atomic packing fraction: $\frac{\pi}{3\sqrt{2}} \approx .74$
  - Face-Centered Cubic
  - Hexagonal Close-Packed



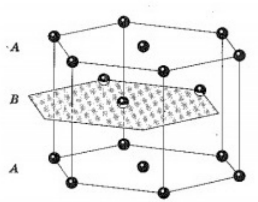


Figure : Face-Centered Cubic (left) and Hexagonal Close-Packed (right) crystal structures. Images are from Kittel's "Introduction to Solid State Physics."

Conclusion

- Primary focus: baryon interactions at sub-leading order
  - Phenomena at different orders in N_c typically do not affect each other
- Nuclear matter saturates
  - Nearest-neighbor distance $\tilde{d} = \frac{1}{c_2 - \tilde{m}_{\text{gb}}} \ln(\frac{c_1 N_c M_q \tilde{\alpha}_s}{\tilde{g}_{\text{gb}}})$
  - Result depends solely upon quantities that are either known or can be computed via lattice QCD
  - Correction term to $\tilde{d}$: $(p+1) \ln \tilde{d}$
  - FCC or HCP Crystal Structure

This research lays a foundation from which more realistic physical situations in QCD may be studied.

Future Investigation

- Calculation of nuclear matter interaction energy
- Identifying whether FCC or HCP is more energetically favorable
  - Purely a theoretical question, as nuclear matter does not generally crystallize
- Lowering N_c and M_q into more realistic regimes
  - A phase transition is expected upon lowering M_q

Acknowledgments

Thank you to Dr. Thomas Cohen, Mr. Prabal Adhikari, Mrs. Angelique Bosse, Mr. James Schafer, and my parents, Soma and Subhasis Datta, for their guidance and support.