University Name

MAT 999:
Course Title

Professor: Instructor Name
Notes By: Note Taker

Semester Year

Last Updated: August 29, 2019

Contents

0 Introduction

0.1 Fermat's Last Theorem

There are many famous theorems in Mathematics. One of the most famous theorems is Fermat's Last Theorem.

Theorem 0.1 (Fermat's Last Theorem)**.** *If $n > 2$, there are no integers a, b, c with $abc \neq 0$ such that $a^n + b^n = c^n$.*

0.2 Purpose

Theorem 0.1 is one of the most famous theorems in Mathematics. But most undergraduate students do not learn Fermat's Last Theorem. Instead, many students learn formulas such as:

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \nabla \times \mathbf{F} \cdot dS$$

But Mathematics starts far more simple than that. The first topic in Mathematics that one typically sees is Arithmetic. For instance, students typically will learn "FOIL."

$$(x+y)^2 = x^2 + 2xy + y^2 \tag{0.1}$$

However, (0.1) tends to be a stumbling block for students. Many students will instead claim, incorrectly, that $(x+y)^2 = x^2 + y^2$. The purpose of this course will be to prove Dirichlet's Unit Theorem, which states:

Theorem 1.1 (Dirichlet's Unit Theorem)**.** *Let K be a number field of degree n with r real embeddings and s conjugate pairs of complex embeddings. Then the abelian group $\mathcal{O}_K^\times$ is a finitely generated abelian group with rank $r + s - 1$ and $\mathcal{O}_K^\times \cong \mu_K \times \mathbb{Z}^{r+s-1}$, where μ_K are the roots of unity in $\mathcal{O}_K$.*

However, it will take some time to prove Theorem 1.1.

1 First Topics

Recall that the goal of this course was to prove Dirichlet's Unit Theorem:

Theorem 1.1 (Dirichlet's Unit Theorem)**.** *Let K be a number field of degree n with r real embeddings and s conjugate pairs of complex embeddings. Then the abelian group $\mathcal{O}_K^\times$ is a finitely generated abelian group with rank $r+s-1$ and $\mathcal{O}_K^\times \cong \mu_K \times \mathbb{Z}^{r+s-1}$, where μ_K are the roots of unity in $\mathcal{O}_K$.*

Proof. L.T.R. □

Example 1.1. If $K = \mathbb{Q}$, then $r = 1$ and $s = 0$ so that $r + s - 1 = 0$. Therefore, $\mathcal{O}_\mathbb{Q}^\times = \mathbb{Z}^\times = \{\pm 1\}$. Of course, this is the most trivial possible example. ◁

To learn even more Mathematics, read [Neu99].

References

[Neu99] Jürgen Neukirch. *Algebraic Number Theory (Schappacher, N., trans.)* New York: Springer-Verlag, 1999.